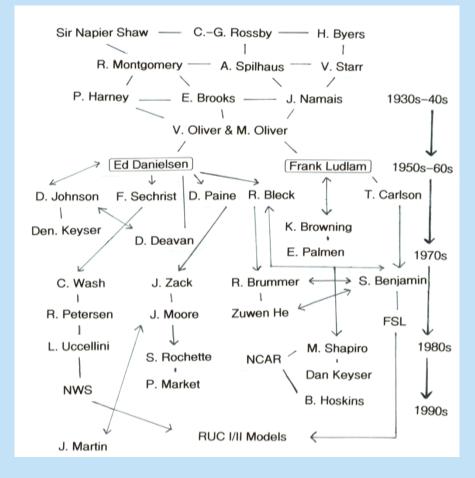
Isentropic Analysis

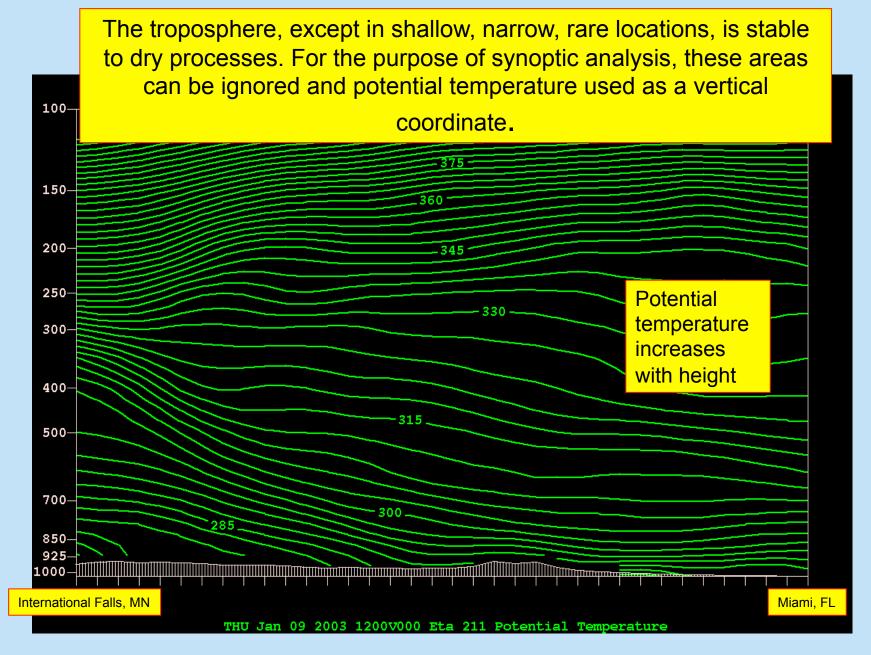
Much of this presentation is due to Jim Moore, SLU



Utility of Isentropic Analysis

- •Diagnose and visualize vertical motion through advection of pressure and system-relative flow
- •Depict three-dimensional transport of moisture
- •Diagnose isentropic potential vorticity (IPV) to depict the dynamic tropopause, tropopause undulations and folds
- •Diagnose upper-level fronts and dry static stability
- •Diagnose potential symmetric instability (PSI)
- •Depict 2-D frontogenetical and transverse jet streak circulations on cross sections

Potential temperature as a vertical coordinate



Potential temperature is conserved during an adiabatic process.

$$\theta = T \left(\frac{1000}{P}\right)^{\frac{R_d}{C_p}}$$

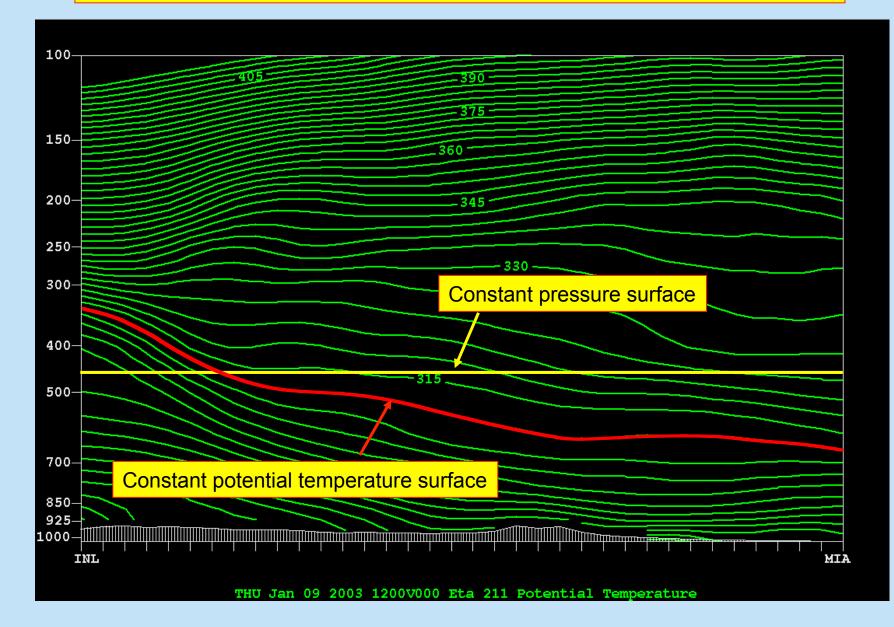
An adiabatic process is isentropic, that is, a process in which entropy is conserved

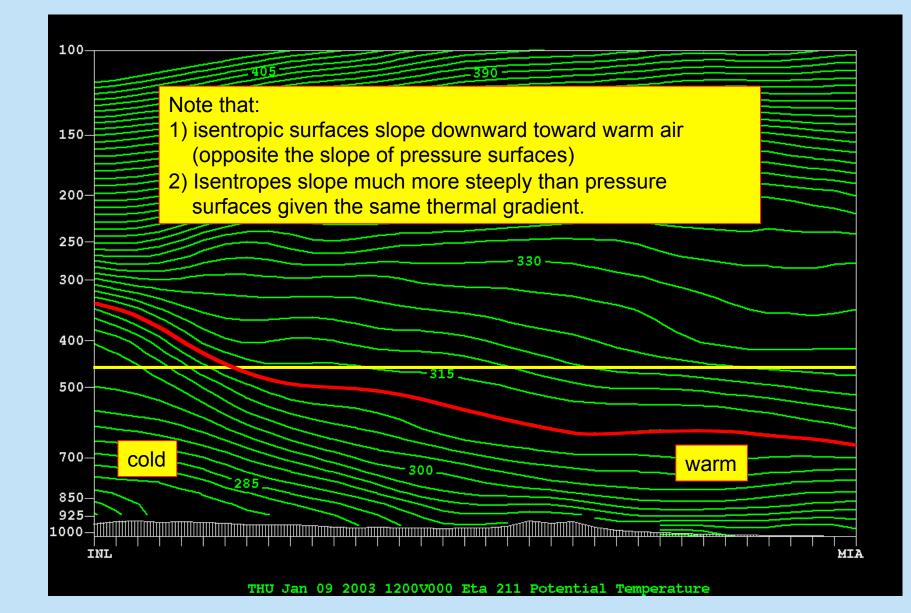
Entropy = $C_p \ln(\theta)$ + constant = 0

Potential temperature <u>is not conserved</u> when 1) diabatic heating or cooling occurs or 2) mixing of air parcels with different properties occurs

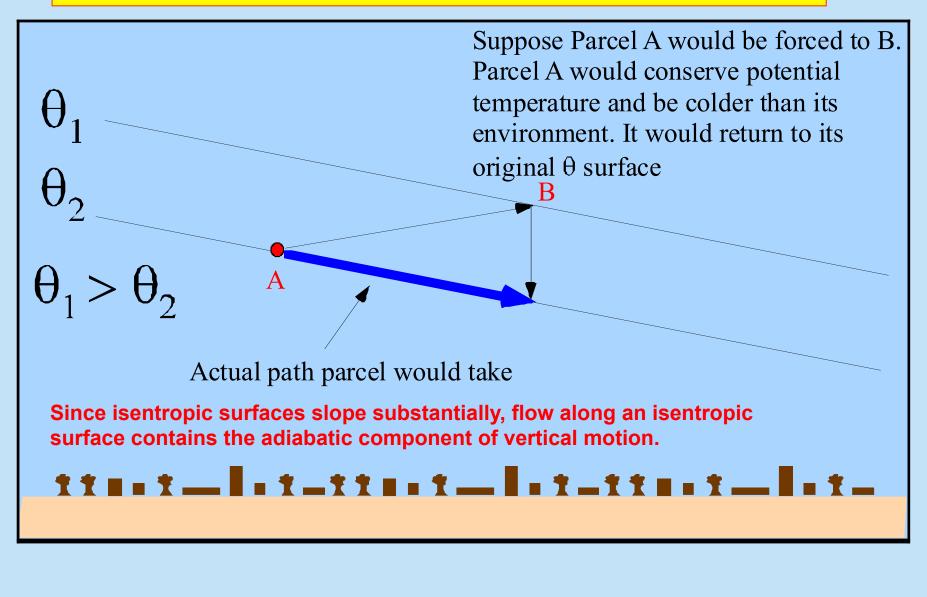
Examples of diabatic processes: condensation, evaporation, sensible heating from surface radiative heating, radiative cooling

Isentropic Analyses are done on constant θ surfaces, rather than constant P or z





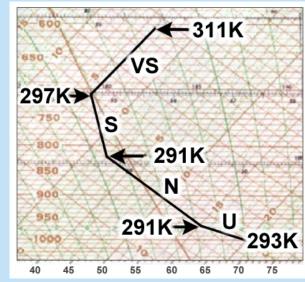
In the absence of diabatic processes and mixing, air flows along θ surfaces. Isentropic surfaces act as "material" surfaces, with air parcels thermodynamically bound to the surface unless diabatic heating or cooling occurs.



Review of Stability and Lapse Rates:

Three types of stability, since ∂θ/∂z = (θ /T) [Γ_d - γ]
-stable: γ < Γ_d, θ increases with height
-neutral: γ = Γ_d, θ is constant with height
-unstable: γ > Γ_d, θ decreases with height

 So, isentropic surfaces are closer together in the vertical in stable air and further apart in less stable air.



Vertical changes of potential temperature related to lapse rates:

U = unstable

N = neutral

VS = very stable

Vertical motion can be expressed as the time derivative of pressure:

$$\omega = \frac{dP}{dt}$$

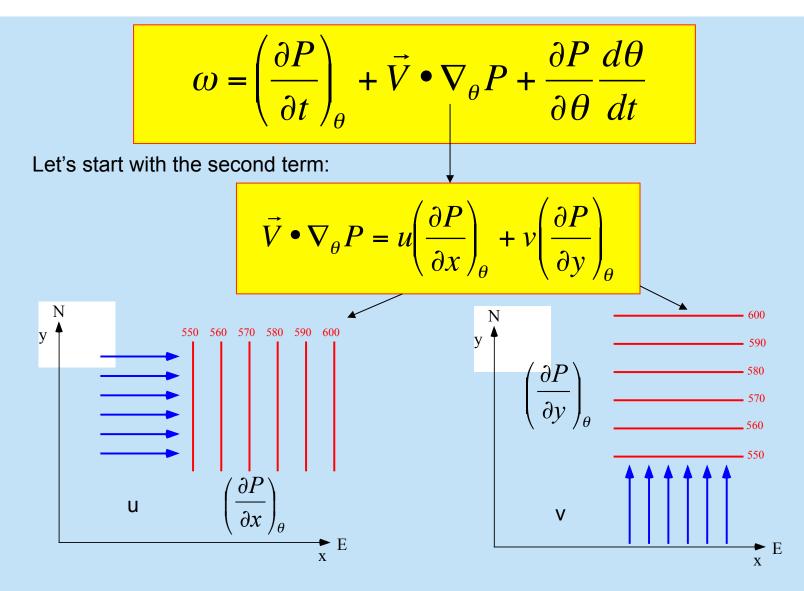
WHEN ω IS POSITIVE, PRESSURE OF AIR PARCEL IS INCREASING WITH TIME – AIR IS DESCENDING.

WHEN ω IS NEGATIVE, PRESSURE OF AIR PARCEL IS DECREASING WITH TIME – AIR IS ASCENDING.

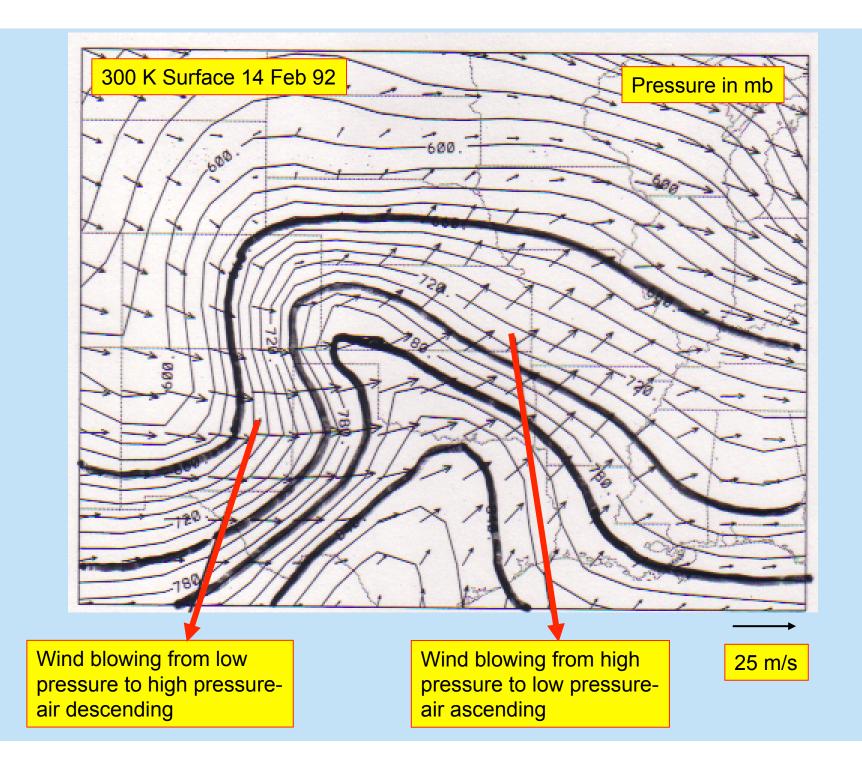
Let's expand the derivative in isentropic coordinates:

$$\omega = \left(\frac{\partial P}{\partial t}\right)_{\theta} + \vec{V} \bullet \nabla_{\theta} P + \frac{\partial P}{\partial \theta} \frac{d\theta}{dt}$$

This equation is an expression of vertical motion in an isentropic coordinate system. Let's look at this equation carefully because it is the key to interpreting isentropic charts



Pressure advection: When the wind is blowing across the isobars on an isentropic chart toward higher pressure, air is descending (ω is positive) When the wind is blowing across the isobars on an isentropic chart toward lower pressure, air is ascending (ω is negative)



Interpretation of "Pressure Advection"

From the equation for θ :

$$\theta = T \left(\frac{1000}{P}\right)^{\frac{R_d}{C_p}}$$

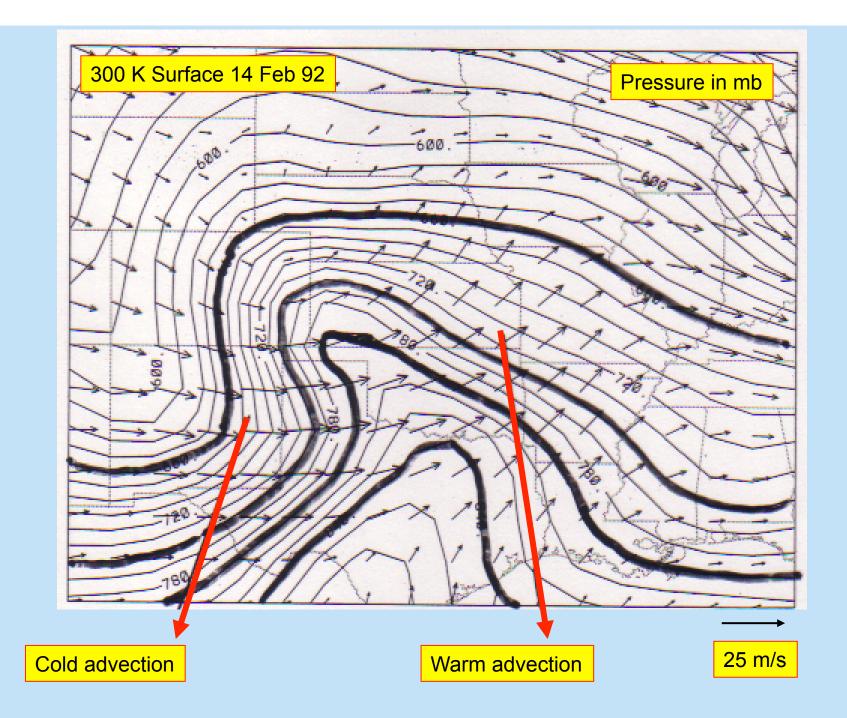
On a constant θ surface, an isobar (line of constant pressure) must also be an isotherm (line of constant temperature)

From the equation of state: $P = \rho RT$ and equation for θ :

On a constant θ surface, an isobar (line of constant pressure) Must also be an isopycnic (line of constant density)

Therefore: On a constant theta surface, pressure advection is equivalent to thermal advection

If wind blows from high pressure to low pressure (ascent): Warm advection If wind blows from low pressure to high pressure (descent): Cold advection



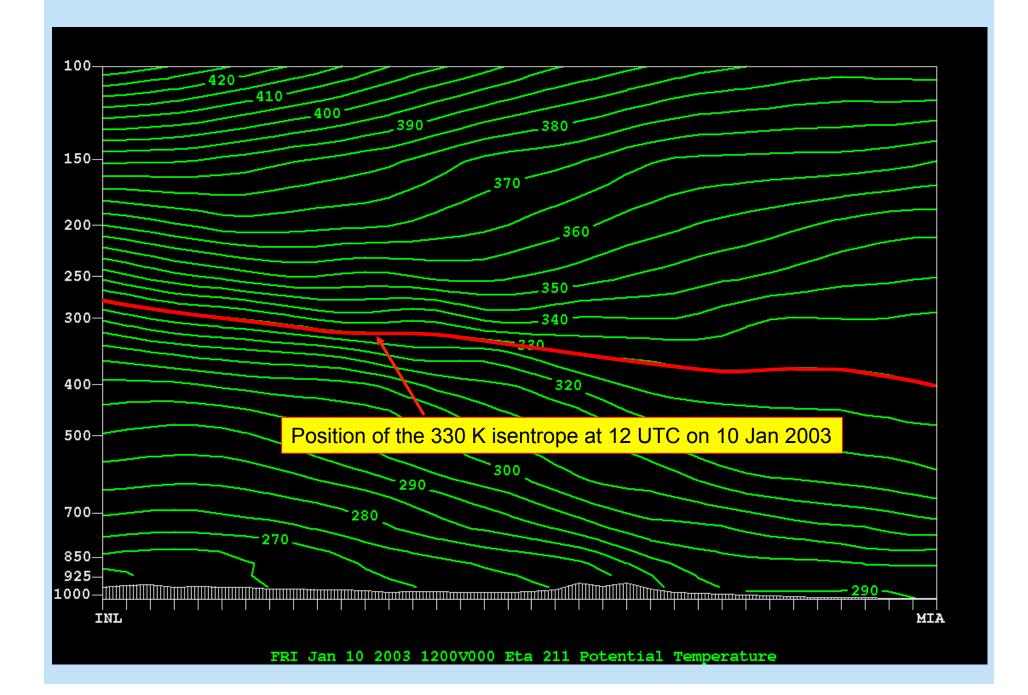
$$\omega = \left(\frac{\partial P}{\partial t}\right)_{\theta} + \vec{V} \cdot \nabla_{\theta} P + \frac{\partial P}{\partial \theta} \frac{d\theta}{dt}$$

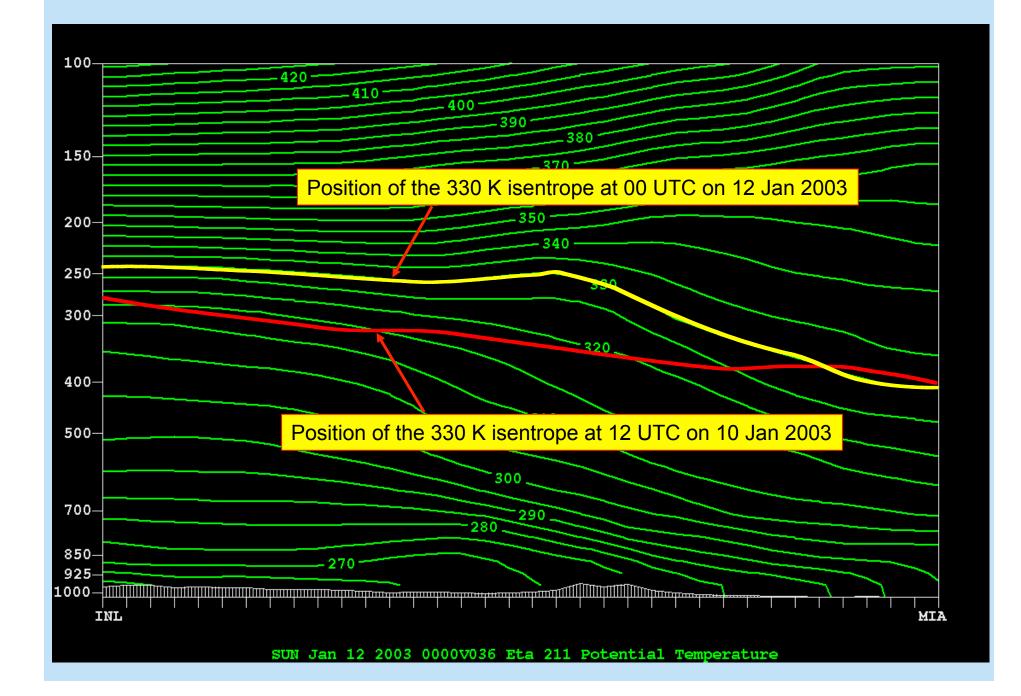
Let's now look at the first term contributing to vertical motion:

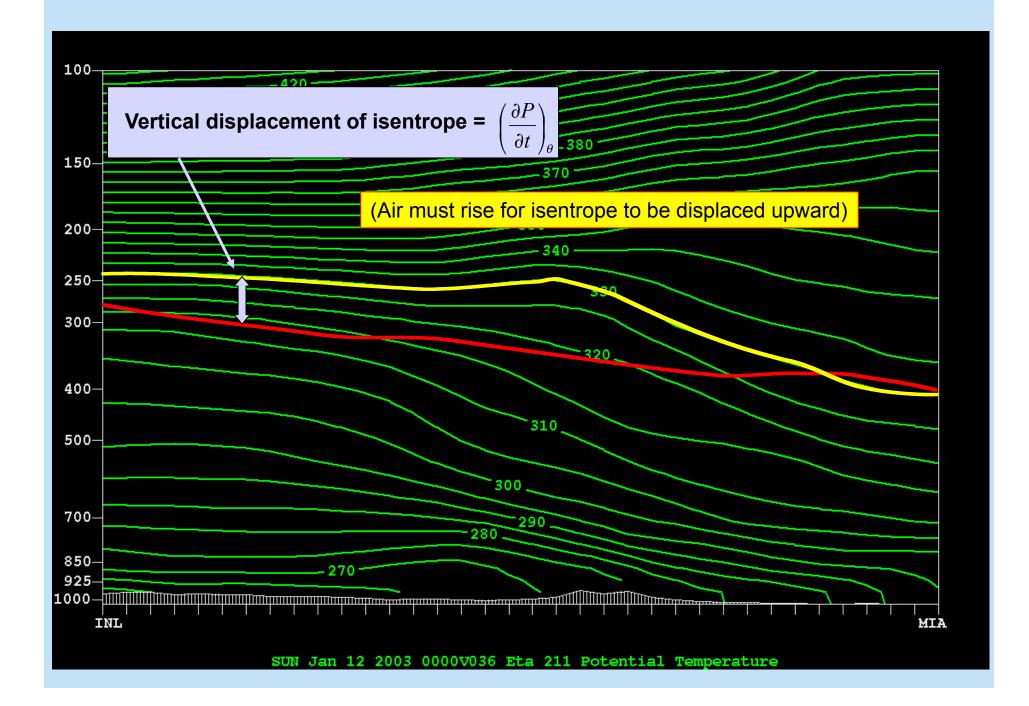
$$\left(\frac{\partial P}{\partial t}\right)_{\theta}$$
 is the local (at one point in x, y) rate of change of pressure of the theta surface with time.

If the theta surface rises, $\left(\frac{\partial P}{\partial t}\right)_{\theta}$ is negative since the pressure at a point on the theta surface is decreasing with time.

If the theta surface descends, $\left(\frac{\partial P}{\partial t}\right)_{\theta}$ is positive since the pressure at a point on the theta surface is increasing with time.

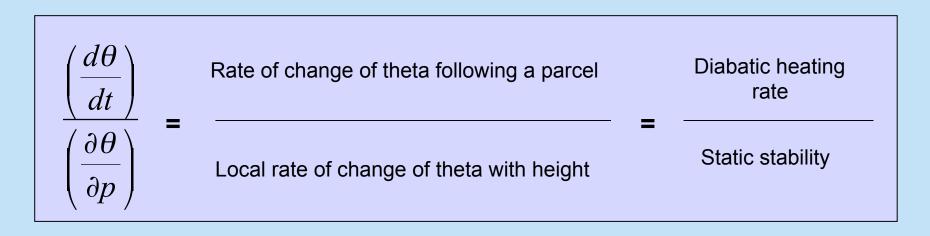






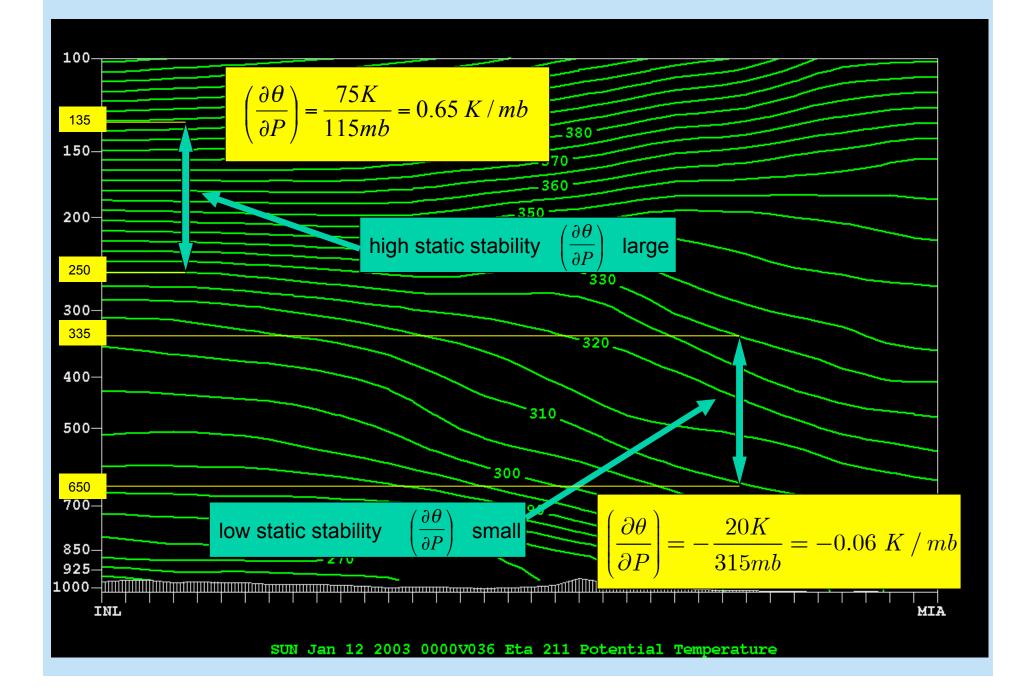
$$\omega = \left(\frac{\partial P}{\partial t}\right)_{\theta} + \vec{V} \cdot \nabla_{\theta} P + \frac{\partial P}{\partial \theta} \frac{d\theta}{dt}$$

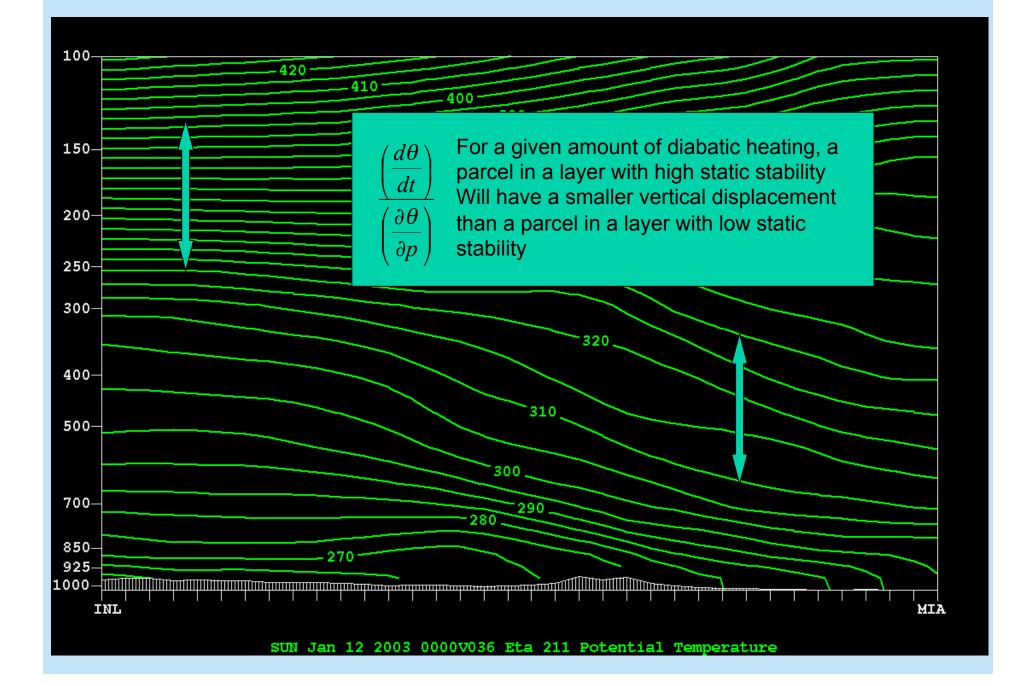
Let's now look at the third term contributing to vertical motion:



Diabatic heating rate = rate that an air parcel is heated (or cooled) by:

Latent heat release during condensation, freezing Latent heat extraction during evaporation, sublimation Radiative heating or cooling





$$\omega = \left(\frac{\partial P}{\partial t}\right)_{\theta} + \vec{V} \cdot \nabla_{\theta} P + \frac{\partial P}{\partial \theta} \frac{d\theta}{dt}$$

Summary:

3rd and 2nd term act in same direction for ascending air: latent heat release will accentuate rising motion in regions of positive pressure advection (warm advection).

3rd term is unimportant in descending air unless air contains cloud or precipitation particles. In this case 3rd term accentuates descending motion in regions of cold advection.

Typical isentropic analyses of pressure only show the second term. This term represents only part of the vertical motion and may be offset (or negated) by 1st term.

Example of Computing Vertical Motion

1. Assume isentropic surface descends as it is warmed by latent heating (local pressure tendency term):

 $\partial P / \partial t = 650 - 550 \text{ hPa} / 12 \text{ h} = +2.3 \text{ µbars s}^{-1} \text{ (descent)}$

2. Assume 50 knot wind is blowing normal to the isobars from high to low pressure (advection term):

 $V \cdot \nabla P = (25 \text{ m s}^{-1}) \text{ x} (50 \text{ hPa}/300 \text{ km}) \text{ x} \cos 180^{\circ}$

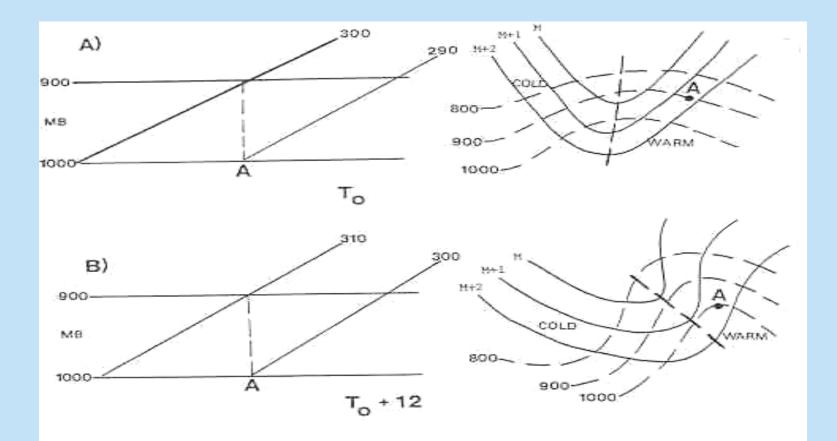
 $\mathbf{V} \cdot \nabla \mathbf{P} = -4.2 \ \mu \text{bars s}^{-1}$ (ascent)

3. Assume 7 K diabatic heating in 12 h in a layer where θ increases 4 K over 50 hPa (diabatic heating/cooling term):

 $(d\theta/dt)(\partial P/\partial \theta) = (7 \text{ K}/12 \text{ h})(-50 \text{ hPa}/4\text{K}) = -2 \mu \text{bars s}^{-1}$

(ascent)

Accounting somewhat for the first term: Understanding System-Relative Motion



Local rate of change term may be impacted by movement of systems - can impact omega calculations

Vertical motion using system relative winds

$$\omega = \left(\frac{\partial P}{\partial t}\right)_{\theta} + \vec{V} \cdot \nabla_{\theta} P + \frac{\partial P}{\partial \theta} \frac{d\theta}{dt}$$
Neglect diabatic processes
$$\omega = \left(\frac{\partial P}{\partial t}\right)_{\theta} + \vec{V} \cdot \nabla_{\theta} P$$
Add and subtract advection
by system
$$\omega = \left(\frac{\partial P}{\partial t}\right)_{\theta} + \vec{V} \cdot \nabla_{\theta} P + \vec{C} \cdot \nabla_{\theta} P - \vec{C} \cdot \nabla_{\theta} P$$
Rearrange
$$\omega = \left(\frac{\partial P}{\partial t}\right)_{\theta} + \vec{C} \cdot \nabla_{\theta} P + (\vec{V} - \vec{C}) \cdot \nabla_{\theta} P$$

System tendency: If the system is neither deepening, filling, or reorienting, term 1 will balance 2 in yellow box so that yellow box = 0

Add

$$\omega^* = \left(\vec{V} - \vec{C}\right) \bullet \nabla_{\theta} P$$

System Relative Vertical Motion

Defined as:

$$\omega^* = \left(\vec{V} - \vec{C}\right) \bullet \nabla_{\theta} P$$

Where ω^* = system-relative vertical motion in µbars sec⁻¹

- V = wind velocity on the isentropic surface
- C = system velocity, and

 $\nabla_{\theta} P$ = pressure gradient on the isentropic surface

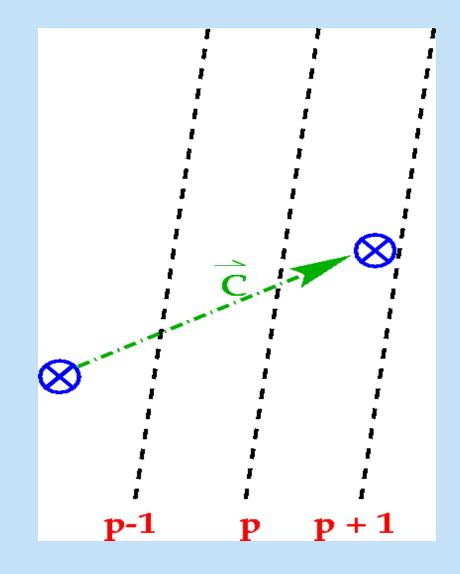
C is computed by tracking the associated vorticity maximum on the isentropic surface over the last 6 or 12 hours (one possible method); another method would be to track the motion of a short-wave trough or cyclone on the isentropic surface Including C, the speed of the system, is important when:

* the system is moving quickly and/or

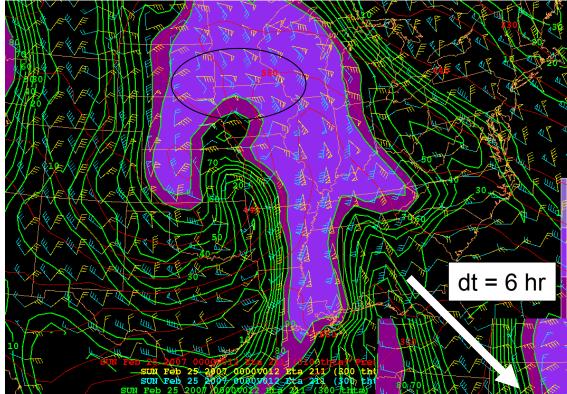
* a significant component of the system motion is **across** the isobars on an isentropic surface,

e.g., if the system motion is from SW-NE and the isobars are oriented N-S with lower pressure to the west, subtracting C from V is equivalent to "adding" a NE wind, thereby increasing the isentropic upslope.

When is C important to use when computing isentropic omegas?



In regions of isentropic upglide, this systemrelative motion vector, C, will enhance the uplift (since C is subtracted from the Velocity vector)



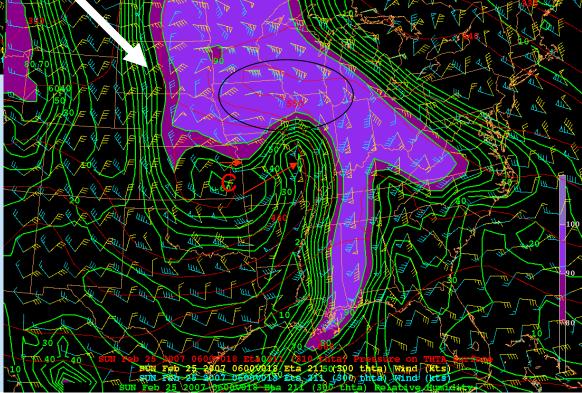
$$\omega^* = \left(\vec{V} - \vec{C}\right) \bullet \nabla_\theta P$$

Estimate storm motion (WNW) $C_i = 400 \text{ km} / 6 \text{ hr} = + 18.5 \text{ m/s}$ $C_i = 50 \text{ km} / 6 \text{ hr} = + 2.3 \text{ m/s}$

& subtract from wind field at each point

Blue barb is observed wind, Yellow barb has system motion subtracted

In GARP, you can enter VSUB(OBS,VECN(C_i, C_j)) to compute storm relative motion on isentropic surfaces.

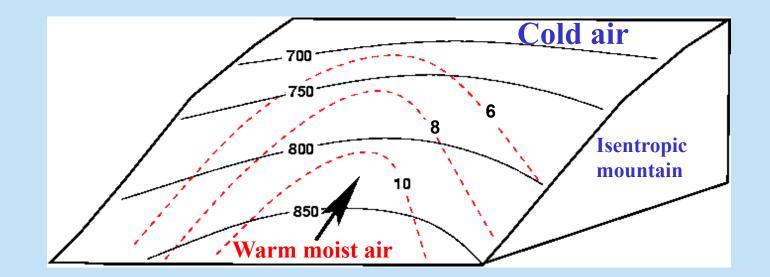


Computing Isentropic Omegas

Essentially there are three approaches to computing isentropic omegas:

- Ground-Relative Method (V ∇P) :
 - Okay for slow-moving systems ($\partial P/\partial t$ term is small)
 - Assumes that the advection term dominates (not always a good assumption)
- System-Relative Method ($(V-C) \bullet \nabla P$):
 - Good for situations in which the system is not deepening or filling rapidly
 - Also useful when the time step between map times is large (e.g., greater than 3 hours)
- Brute-Force Computational Method (∂ P/ ∂ t + V ∇P):
 - Best for situations in which the system is rapidly deepening or filling
 - Good approximation when data are available at 3 h or less interval, allowing for good estimation of local time tendency of pressure

Transport of Moisture on an Isentropic Surface



- - - Mixing ratio contours

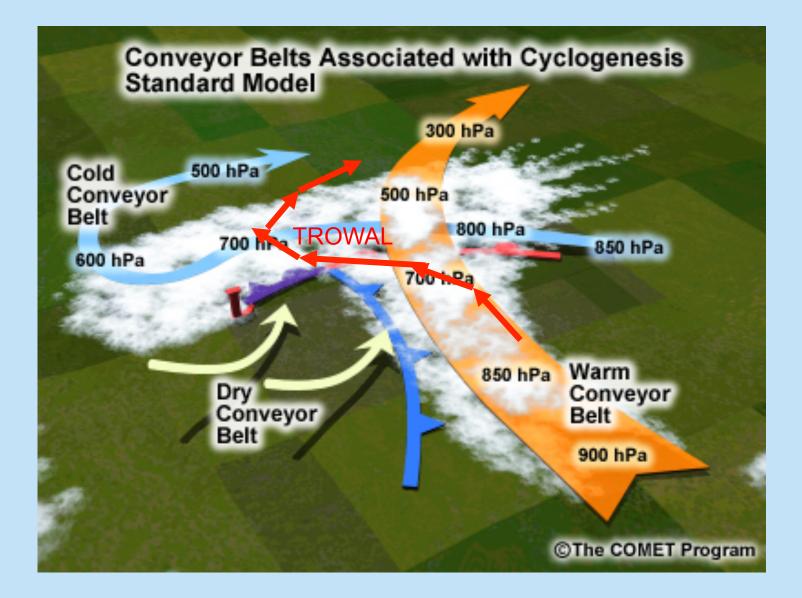
System-Relative Flow: How can it be used?

Carlson (1991, Mid-Latitude Weather Systems) notes:

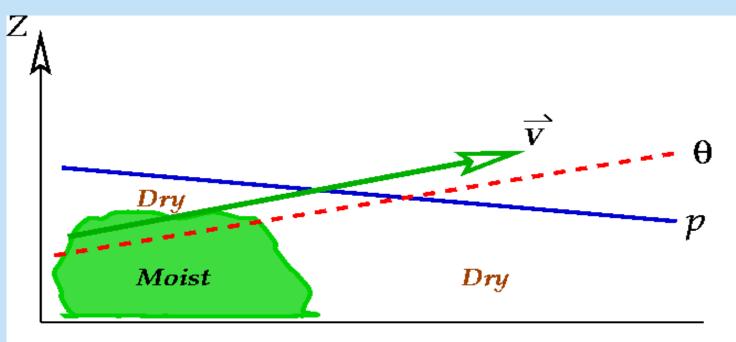
• Relative-wind analysis reveals the existence of sharplydefined boundaries, which differentiate air streams of vastly differing moisture contents. Air streams tend to contain relatively narrow ranges of θ and θ e peeculiar to the air stream's origins.

 Relative-wind isentropic analyses for synoptic-scale weather systems tend to show well-defined air streams, which have been identified by the names – warm conveyor belt, cold conveyor belt and dry conveyor belt.

• Thus, a conveyor belt consists of an ensemble of air parcels having nearly the same θ (or θ_e) value, starting from a common initial location, which travel over synoptic-scale time periods (> 18 h).



Transport of Moisture on an Isentropic Surface

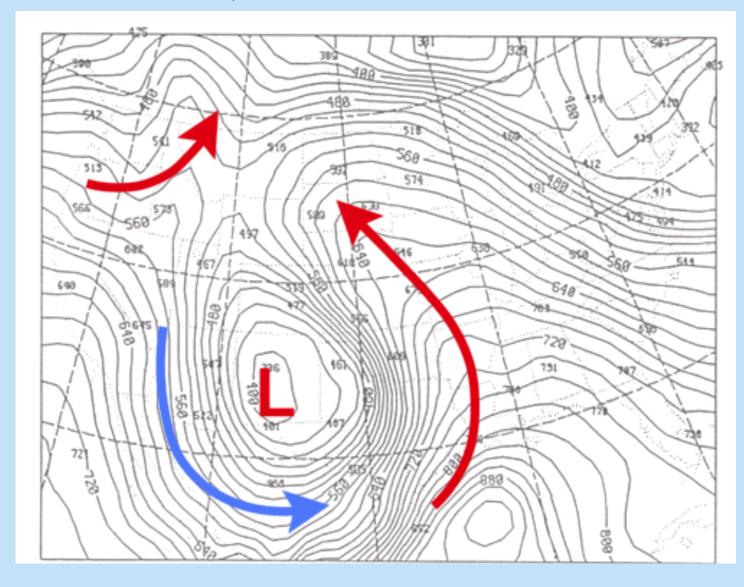


South

North

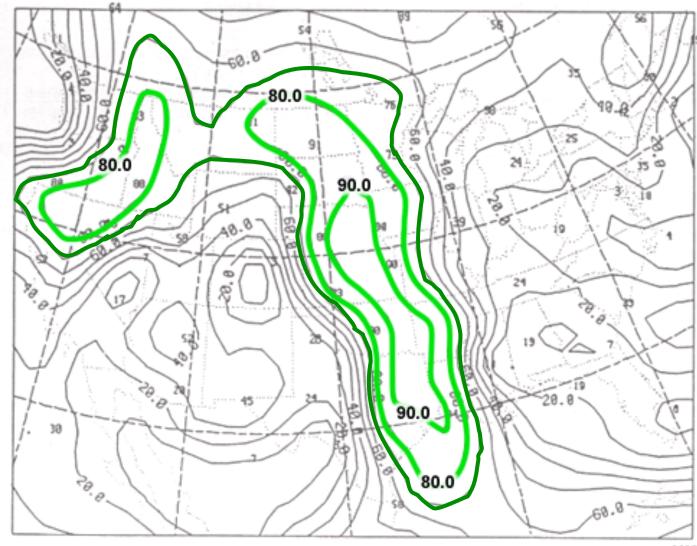
Moist air from low levels on the left (south) is transported **upward** and to the right (north) along the isentropic surface. However, in pressure coordinates water vapor appears on the constant pressure surface labeled p in the absence of advection along the pressure surface --it appears to come from nowhere as it emerges from another pressure surface. (adapted from Bluestein, vol. I, 1992, p. 23)

Pressure analysis 305K surface 12 UTC 3-17-87



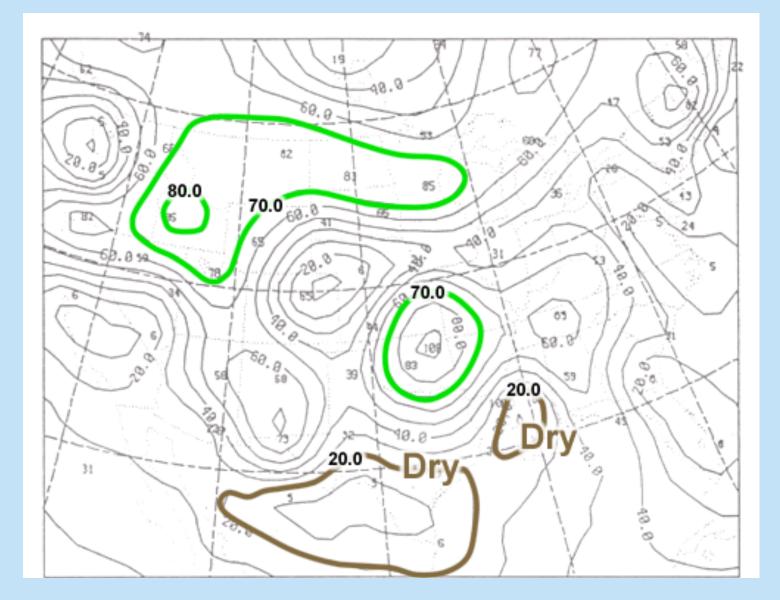
From :Benjamin et al. 1988

Relative Humidity 305K surface 12 UTC 3-17-87 RH>80% = green

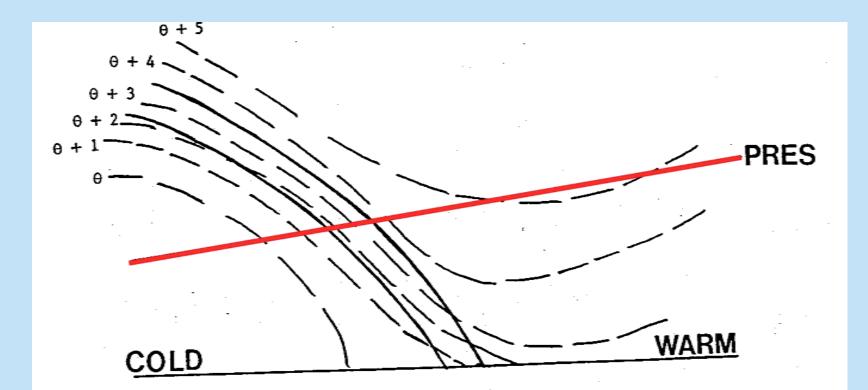


AMS

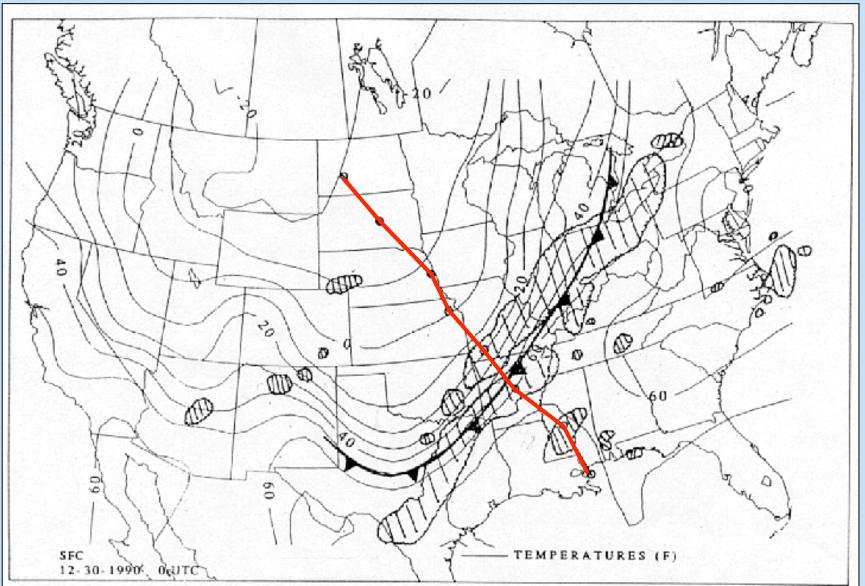
Relative Humidity at 500 hPa RH > 70% = green



Isentropes near Frontal Zones

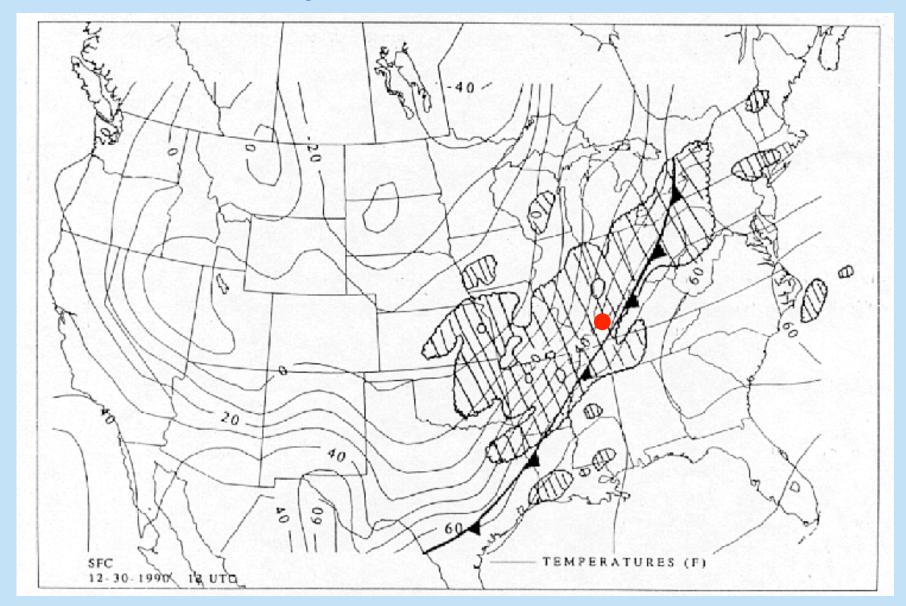


Frontal regions are characterized by high static stability

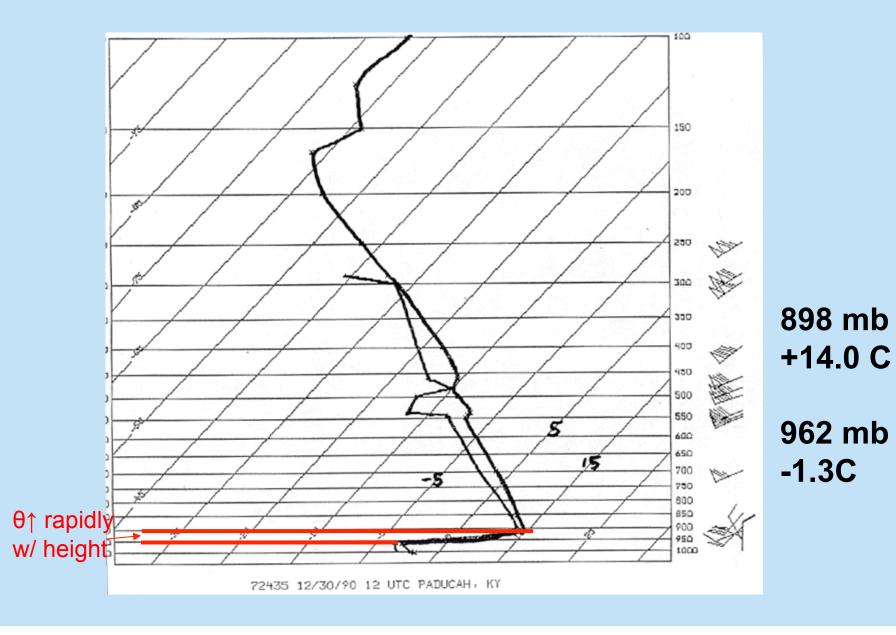


Surface Map for 00 UTC 30 December 1990

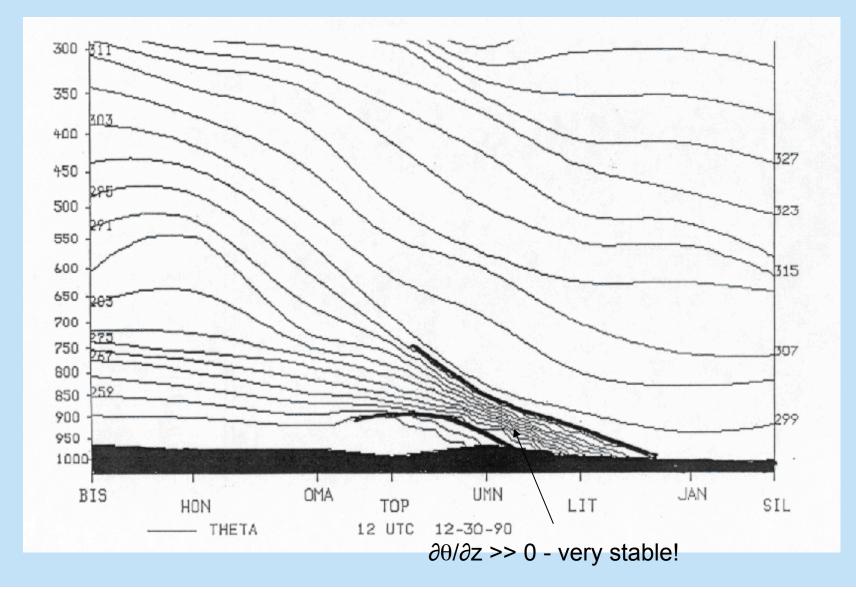
Surface Map for 12 UTC 30 December 1990



Sounding for Paducah, KY 30 December 1990 12 UTC

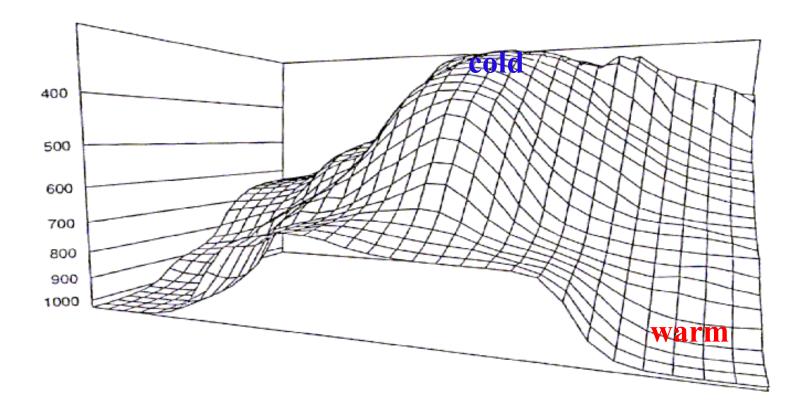


Cross Section Taken Normal to Arctic Frontal Zone: 12 UTC 30 December 1990



Three-Dimensional Isentropic Topography

300 K Isentropic Surface



Representation of the "pressure gradient" on an isentropic surface

Consider the differential of pressure on a surface of constant "s" where "s" is any other coordinate

$$\left(dP\right)_{s} = \left(\frac{\partial P}{\partial x}\right)_{y,z} dx + \left(\frac{\partial P}{\partial y}\right)_{x,z} dy + \left(\frac{\partial P}{\partial z}\right)_{x,y} dz$$

Let us now use $s = \theta$

$$\left(dP \right)_{\theta} = \left(\frac{\partial P}{\partial x} \right)_{z,y} dx + \left(\frac{\partial P}{\partial y} \right)_{z,x} dy + \left(\frac{\partial P}{\partial z} \right)_{x,y} dz$$

Let's consider a pressure gradient in the x direction only (dy = 0).

Then:

$$(dP)_{\theta} = \left(\frac{\partial P}{\partial x}\right)_{z,y} dx + \left(\frac{\partial P}{\partial z}\right)_{x,y} dz$$

From last page:

$$(dP)_{\theta} = \left(\frac{\partial P}{\partial x}\right)_{z,y} dx + \left(\frac{\partial P}{\partial z}\right)_{x,y} dz$$

Divide by dx:

$$\left(\frac{dP}{\partial x}\right)_{\theta} = \left(\frac{\partial P}{\partial x}\right)_{z} + \left(\frac{\partial P}{\partial z}\right)_{x} \left(\frac{dz}{dx}\right)_{\theta}$$

Substitute hydrostatic equation $\left(\frac{\partial P}{\partial z}\right)_r = -g\rho$ and divide by - ρ : $-\frac{1}{\rho} \left(\frac{dP}{\partial x} \right)_{\theta} = -\frac{1}{\rho} \left(\frac{\partial P}{\partial x} \right)_{z} + g \left(\frac{dz}{dx} \right)_{\theta}$

Let's find an expression for $\left(\frac{dP}{\partial x}\right)_{\theta}$

$$\theta = T \left(\frac{1000}{P}\right)^{\frac{R_d}{C_p}}$$
Solve for P
$$P = \frac{P_0}{\theta^{C_p/R_d}} T^{C_p/R_d} \qquad (1)$$
Take $\left(\frac{\partial P}{\partial x}\right)_{\theta} = \left(\frac{\partial P}{\partial x}\right)_{\theta} = \left(\frac{P_0}{\theta^{C_p/R_d}}\right) \left(\frac{C_p}{R_d}\right) (T^{(C_p/R_d)-1}) \left(\frac{\partial T}{\partial x}\right)_{\theta}$

$$\left(\frac{\partial P}{\partial x}\right)_{\theta} = P\left(\frac{C_p}{R_d}\right) \left(\frac{T^{(C_p/R_d)-1}}{T^{(C_p/R_d)}}\right) \left(\frac{\partial T}{\partial x}\right)_{\theta}$$

Use Ideal Gas law to replace P

Substitute from (1)

$$\left(\frac{\partial P}{\partial x}\right)_{\theta} = \rho R_d T \left(\frac{C_p}{R_d}\right) \left(\frac{T^{(C_p/R_d)-1}}{T^{(C_p/R_d)}}\right) \left(\frac{\partial T}{\partial x}\right)_{\theta}$$

From last page:

$$\begin{pmatrix} \frac{\partial P}{\partial x} \end{pmatrix}_{\theta} = \rho R_d T \left(\frac{C_p}{R_d} \right) \left(\frac{T^{(C_p/R_d)-1}}{T^{(C_p/R_d)}} \right) \left(\frac{\partial T}{\partial x} \right)_{\theta}$$
Cancel temperatures and R_d

$$\begin{pmatrix} \frac{\partial P}{\partial x} _{\theta} \end{pmatrix}_{\theta} = C_p \rho \left(\frac{\partial T}{\partial x} \right)_{\theta}$$
Remember this?

$$-\frac{1}{\rho} \left(\frac{dP}{\partial x} \right)_{\theta} = -\frac{1}{\rho} \left(\frac{\partial P}{\partial x} \right)_z + g \left(\frac{dz}{dx} \right)_{\theta}$$
Let's find an expression for $\left(\frac{dP}{\partial x} \right)_{\theta}$

$$-\frac{1}{\rho} \left(C_P \rho \left(\frac{\partial T}{\partial x} \right)_{\theta} \right) = -\frac{1}{\rho} \left(\frac{\partial P}{\partial x} \right)_z + g \left(\frac{dz}{dx} \right)_{\theta}$$

$$-\frac{1}{\rho} \left(C_p \rho \left(\frac{\partial T}{\partial x} \right)_{\theta} \right) = -\frac{1}{\rho} \left(\frac{\partial P}{\partial x} \right)_z + g \left(\frac{dz}{dx} \right)_{\theta}$$

Rearrange:
$$-\frac{1}{\rho} \left(\frac{\partial P}{\partial x} \right)_z = -\left(\frac{d}{dx} \left(C_p T + gz \right) \right)_{\theta} \qquad \boxed{M = C_p T + gz}$$

Or:
$$-\frac{1}{\rho} \left(\frac{\partial P}{\partial x} \right)_z = -\left(\frac{dM}{dx} \right)_{\theta}$$

Similarly:
$$-\frac{1}{\rho} \left(\frac{\partial P}{\partial y} \right)_z = -\left(\frac{dM}{dy} \right)_{\theta}$$

R

S

M is called the Montgomery Streamfunction

The pressure gradient force on a constant height surface is equivalent to the gradient Of the Montgomery streamfunction on a constant potential temperature surface.

Therefore: Plots of M on a potential temperature surface can be used to illustrate the pressure gradient force, and **thus the direction and speed of the geostrophic flow.**

Depicting geostrophic flow on an isentropic surface:

On a pressure surface, the geostrophic flow is depicted by height contours, where the geostrophic wind is parallel to the height contours, and its magnitude is proportional to the spacing of the contours

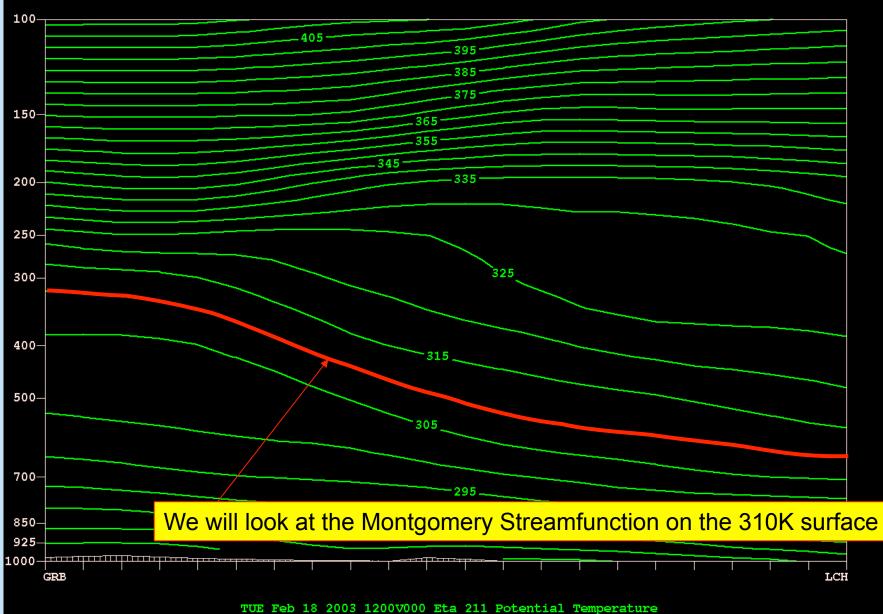
On an isentropic surface, the geostrophic flow is depicted by contours of the Montgomery streamfunction, where the geostrophic wind is parallel to the contours of the Montgomery Streamfunction and is proportional to their spacing.

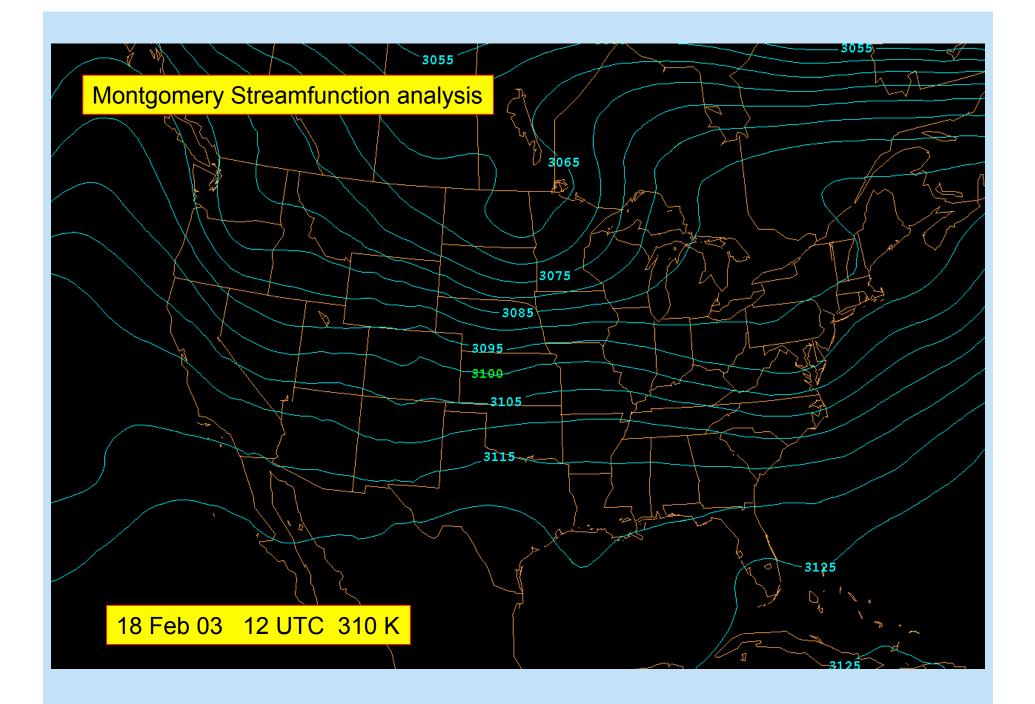
$$M = C_p T + gz$$

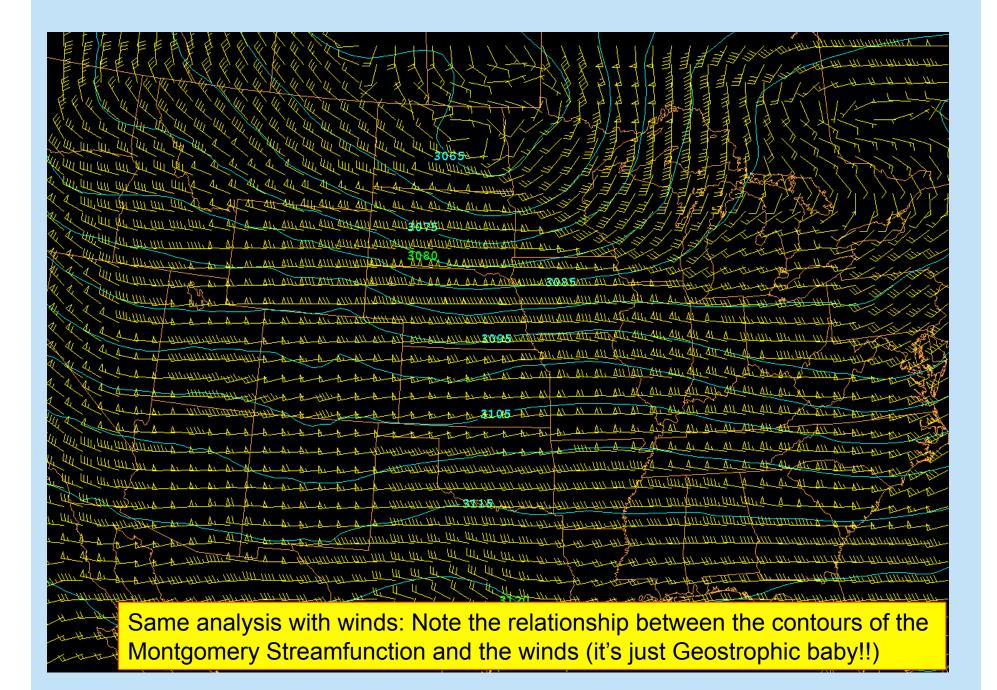
Since $-\frac{1}{\rho} \left(\frac{\partial P}{\partial y} \right)_{z} = -\left(\frac{dM}{dy} \right)_{\theta}$ and $\mathbf{V}_{g} = \frac{1}{f\rho} \hat{\mathbf{k}} \times \nabla_{z} p$

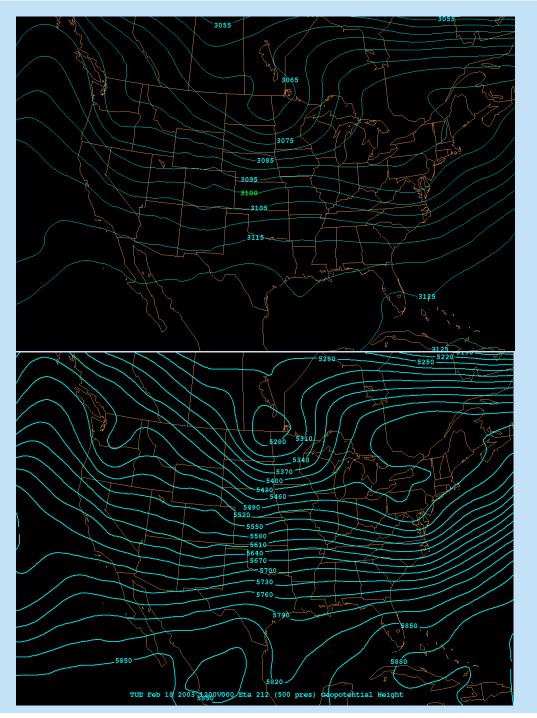
It follows that

 $\mathbf{V}_{g} = \frac{1}{f} \mathbf{\hat{k}} \times \nabla_{\theta} M$









Montgomery Streamfunction 310 K

(Plot in GARP using the variable PSYM)

Height contours 500 mb

Conservative variables on isentropic surfaces:

Mixing ratio

Use mixing ratio to determine moisture transport and RH to determine cloud patterns

Isentropic Potential Vorticity

$$P = -g(\zeta_{\theta} + f)\frac{\partial\theta}{\partial p}$$

Where:
$$\zeta_{\theta} = \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)_{\theta}$$

Isentropic potential vorticity is of the order of:

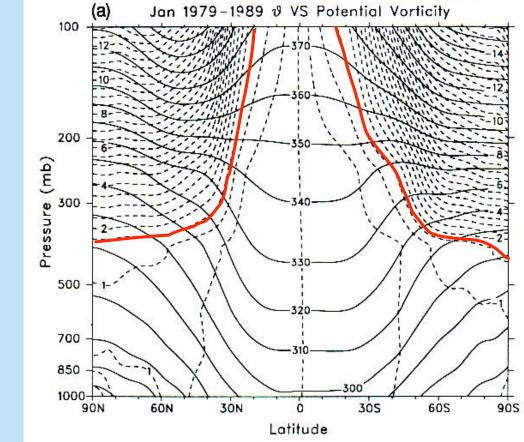
$$P = -g(\zeta_{\theta} + f)\frac{\partial\theta}{\partial p} \approx (10\,m\,s^{-2})(10^{-4}\,s^{-1})\left(\frac{10K}{10kPa}\right)\frac{1\,kPa}{10^{3}\,kg\,m\,s^{-2}m^{-2}}$$

$$P = 10^{-6} m^2 s^{-1} K kg^{-1} = 1 PVU$$

Isentropic Potential Vorticity

Values of IPV < 1.5 PVU are generally associated with tropospheric air

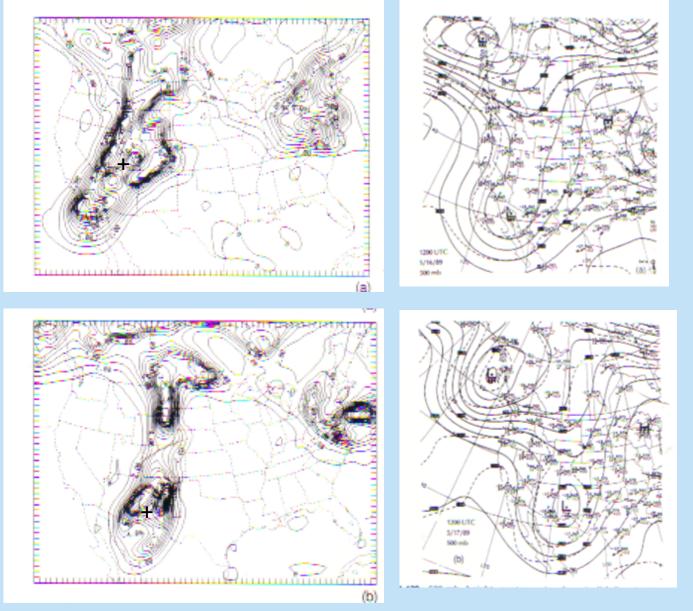
Values of IPV > 1.5 PVU are generally associated with stratospheric air



Global average IPV in January

Note position of IPV =1.5 PVU

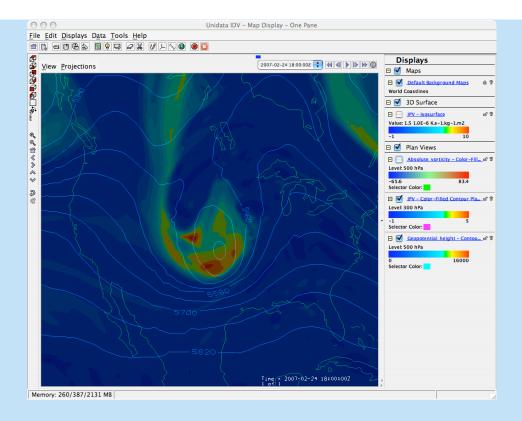
Fig. 1.137 Bluestein II



Relationship Between IPV Distribution on The 325 K surface And 500 mb height contours

12Z May 16 1989

12Z May 17 1989



Regions of relatively high PV are called "positive PV anomalies"

These are associated with cyclonic circulations and low static stability in the troposphere

Regions of relatively low PV are called "negative PV anomalies"

These are associated with anticyclonic circulations and high static stability in the troposphere

For adiabatic, inviscid (no mixing/friction) flow, IPV is a conservative tracer of flow.

IPV has come into use in forecasting because of its invertibility:

If one assumes a balance condition (e.g. geostrophic balance, gradient wind balance or higher order balances), specification of the IPV field allows one to deduce the pressure and wind fields. *Why do these assumptions allow you to use only IPV to determine the state of the atmosphere?*

We will learn more about IPV later on in this course!

$$P = -g(\zeta_{\theta} + f)\frac{\partial\theta}{\partial p}$$

Isentropic Analysis: Advantages



For synoptic scale motions, in the absence of diabatic processes, isentropic surfaces are material surfaces,
i.e., parcels are thermodynamically bound to the surface.

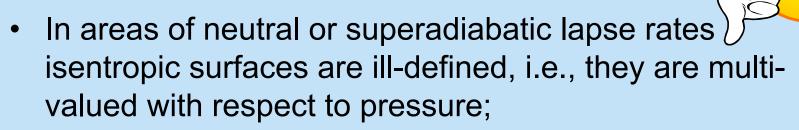
- Horizontal flow along an isentropic surface contains the adiabatic component of vertical motion.
- Moisture transport on an isentropic surface is threedimensional - patterns are more spatially and temporally coherent than on pressure surfaces.
- Isentropic surfaces tend to run parallel to frontal zones making the variation of basic quantities (u,v, T, q) more gradual along them.

Isentropic Analysis: Advantages



- Atmospheric variables tend to be better correlated along an isentropic surface upstream/downstream, than on a constant pressure surface, especially in advective flow
- The vertical spacing between isentropic surfaces is a measure of the dry static stability. Divergence (convergence) between two isentropic surfaces decreases (increases) the static stability in the layer.
- The slope of an isentropic surface (or pressure gradient along it) is directly related to the thermal wind.
- Parcel trajectories can easily be computed on an isentropic surface. Lagrangian (parcel) vertical motion fields are better correlated to satellite imagery than Eulerian (instantaneous) vertical motion fields.

Isentropic Analysis: Disadvantages



- In areas of near-neutral lapse rates there is poor vertical resolution of atmospheric features. In stable frontal zones, however there is excellent vertical resolution.
- Diabatic processes significantly disrupt the continuity of isentropic surfaces. Major diabatic processes include: latent heating/evaporative cooling, solar heating, and infrared cooling.
- Isentropic surfaces tend to intersect the ground at steep angles (e.g., SW U.S.) require careful analysis there.

Isentropic Analysis: Disadvantages



•The "proper" isentropic surface to analyze on a given day varies with season, latitude, and time of day. There are no fixed levels to analyze (e.g., 500 hPa) as with constant pressure analysis.

•If we practice "meteorological analysis" the above disadvantage turns into an advantage since we must think through what we are looking for and why!